1	**(i)**		$x = 10t$ $y = 10\sqrt{3}t - 4.9t^2$	**B1** **B1** **[2]**	Allow $x = 20\cos 60^\circ\, t$ Allow $y = 20\sin 60^\circ\, t - \frac{g}{2}t^2$ or $y = 17.3\, t - \frac{9.8}{2}t^2$	
	(ii)		Substitute $t = \frac{x}{10}$ in equation for y $\Rightarrow y = \sqrt{3}x - 0.049x^2$	**M1** **A1** **[2]**	Substitution of a correct expression for t. Notice that this is a given result	
	(iii)		When $y = 0$, $x = \frac{1.732}{0.049}$ (or 0) The range is 35.3 m	**M1** **A1** **[2]**	Use of y = 0, or 2 × Time to maximum height	
	(iv)		When $x = 20, y = 1.732 \times 20 - 0.049 \times 20^2$ Height is 15.04 m so passes below the bird whose height is 16 m	**M1** **A1** **[2]**	Use of equation of trajectory **Special Case** Allow **SC2** for substituting y = 16 in the trajectory, showing the equation for x has no real roots and concluding the height of the ball is always less than16 m. This can also be done with the equation for vertical motion.	
	(iv)		**Alternative: Using time** When x = 20, t = 2 $y = 10\sqrt{3} \times 2 - 4.9 \times 2^2$ Height is 15.04 m so passes below the bird whose height is 16 m	 **M1** **A1**	 Use of equation for the height	
	(iv)		**Alternative: Maximum height** The maximum height of the ball (is 15.3 m) Since15.3 < 16, it is always below the bird	 **M1** **A1**	 A valid method for finding the maximum height	

Question			Answer	Marks	Guidance	
2	(i)		Vertical component of initial velocity $= 20\sin 30°$ $(=10)$	B1		
			Vertical motion $s = s_0 + ut + \frac{1}{2}at^2$	M1	Substitution required. The sign of g must be correct. Condone no s_0	
			When it hits the sea $0 = 75 + 10t - 5t^2$	A1		
			$75 + 10 \times 5 - 5 \times 5^2 = 0$ As required		Or equivalent, eg solving the quadratic equation.	
			This is satisfied when $t = 5$	E1		
			Alternative			
			Vertical component of initial velocity $= 20\sin 30°$ $(=10)$	B1		
			Vertical motion $v = u + at$	M1	Complete method for finding $t = 5$ required.	
			At the top $0 = 10 - 10t \Rightarrow t = 1$			
			It takes another 1 second to reach the level of the cliff top			
			At that point its speed is 10 m s^{-1} downwards			
			When it hits the sea $-75 = -10t - 5t^2$		Or equivalent finding the time (4 seconds) from the top (height 80 m) to hitting the sea	
			$t^2 + 2t - 15 = 0 \Rightarrow t = 3$	A1		
			Total time = 1 + 1 + 3 = 5 seconds	E1		
			Horizontal motion $x = 20 \times \cos 30° \times t$	M1		
			$t = 5 \Rightarrow 86.6$			
			It is 3.4 m from the ship so within 5 m	E1	Condone 3.5 m	
				[6]		
	(ii)		It is longer in the air so it goes further	B1	Justification for travelling further is required for this mark.	
				[1]		

Question			Answer	Marks	Guidance
3	(i)		**Either**		
			Both components of initial speed Horiz $31\cos 20°$ (29.1) Vert $31\sin 20°$ (10.6)	B1	No credit if sin-cos interchanged The components may be found anywhere in the question
			Time to goal $= \dfrac{50}{31\cos 20°}$	M1	Attempt to use horizontal distance ÷ horizontal speed
			$= 1.716$ ... s	A1	
			$h = 31 \times \sin 20° \times 1.716 + 0.5 \times (-9.8) \times (1.716)^2$	M1	Use of one (or more) formula(e) to find the required result(s) relating to vertical motion within a correct complete method. Finding the maximum height is not in itself a complete method.
			$h = 3.76$ (m)	A1	Allow 3.74 or other answers that would round to 3.7 or 3.8 if they result from premature rounding
			So the ball goes over the crossbar	E1	Dependent on both M marks. Allow follow through from previous answer
			Or		
			Both components of initial speed	B1	May be found anywhere in the question. No credit if sin-cos interchange
			$h = 31\sin 20° \times t - 4.9t^2$	M1	
			Substitute $h = 2.44 \Rightarrow t =$ (0.26 or) 1.90	A1	If only 0.26 is given, award A0
			Substitute $t = 1.90$ in $x = 31\cos 20° \times t$	M1	Allow this mark for substituting $t = 0.26$
			$x = 55.4$	A1	Allow $x = 7.6$ following on from $t = 0.26$
			Since $55.4 > 50$ the ball goes over the crossbar	E1	Dependent on both M marks. Allow FT from their value for 55.4.
			Or		
			Both components of initial speed	B1	May be found anywhere in the question. No credit if sin-cos interchanged
			$h = 31\sin 20° \times t - 4.9t^2$	M1	
			Substitute $h = 2.44 \Rightarrow t =$ (0.26 or) 1.90	A1	
			Time to goal $= \dfrac{50}{31\cos 20°}$	M1	Attempt to use horizontal distance ÷ horizontal speed
			$= 1.716$ ... s	A1	
			Since $1.90 > 1.72$ the ball goes over the crossbar	E1	Dependent on both M marks. Allow follow through from previous answer

			Or Use of the equation of the trajectory $y = x\tan 20° - \dfrac{9.8x^2}{2\times 31^2 \times \cos^2 20°}$ Substituting $x = 50$ $\Rightarrow y = 3.76$ So the ball goes over the crossbar	 M1 A1 A1 M1 A1 E1	 Correct substitution of $\alpha = 20°$ Fully correct Dependent on both M marks. Follow through from previous answer
3	(ii)		Any one reasonable statement	B1 **[1]**	**Accept** The ground is horizontal The ball is initially on the ground Air resistance is negligible Horizontal acceleration is zero The ball does not swerve There is no wind The particle model is being used The value of *g* is 9.8 **Do not accept** *g* is constant

4		mark	notes
	$v^2 = 11^2 + 2\times(-9.8)\times 2.4$ v = 8.6 so 8.6 m s $^{-1}$.	M1 A1 A1	Use of $v^2 = u^2 + 2as$ or complete sequence of correct *suvat*. Accept sign errors in substitution. All correct cao [Award all marks if 8.6 seen WWW] Do not condone ±8.6.
		3	

5		mark	notes
	Usual notation **either** consider height:		
	Attempt to substitute for u and a in $s = ut + \frac{1}{2}at^2$	M1	Accept: g as g, ±9.8, ±9.81, ±10; u = 30; s ↔ c .
	$y = 30\sin 35\ t - 4.9t^2$	A1	Derivation need not be shown
	Need y = 0 for time of flight T	B1	
	giving $T = \dfrac{30\sin 35}{4.9}$ (= 3.511692…)	A1	cao. Any form. May not be explicit.
	Or Consider time to top Attempt to substitute for u and a in $v = u + at$	M1	Accept: g as g, ±9.8, ±9.81, ±10; u = 30; s ↔ c .
	$v = 30\sin 35 - 9.8t$	A1	Derivation need not be shown
	Need v = 0 and to double for time of flight T	B1	
	giving $T = \dfrac{30\sin 35}{4.9}$ (= 3.511692…)	A1	cao. Any form. May not be explicit.
	then $x = 30\cos 35\ T$	M1	Accept s ↔ c if consistent with above
	so x = $30\cos 35 \times \dfrac{30\sin 35}{4.9}$ (= 86.29830…)	F1	FT for their time Condone consistent s ↔ c error (which could lead to correct answer here).
	Required time for sound is x/343	M1	FT from their x
	Total time is 3.511692… + 0.251598… = 3.76329… so 3.76 s (3 s. f.)	A1	cao following fully correct working throughout question.
		8	

6		mark	notes
(i)	Vertica		
.	$y = 8t - 4.9t^2$	M1	Use of $s = ut + 0.5at^2$ with $g = \pm 9.8,\ \pm 10$. Accept u = 0 or 14.4… or 14.4 $\sin\theta$ or $u\sin\theta$ but not 12 . Allow use of + 3.6.
		A1	Accept derivation of – 4.9 not clear. cao.
	Horizontally $x = 12t$	B1	
		3	
(ii)	**either**		
	Require $y = -3.6$	M1	Equating **their** y to ±3.6 or equiv. Any form.
	so $-3.6 = 8t - 4.9t^2$		
	Use of formula or $4.9(t-2)(t+\frac{18}{49}) = 0$	M1	A method for solving a 3 term quadratic to give at least 1 root. Allow **their** y and re-arrangement errors.
	Roots are 2 and $-\frac{18}{49}$ (= – 0.367346…)	A1	WWW. Accept no reference to 2nd root [Award SC3 for t = 2 seen WWW]
	Horizontal distance is 12 × 2 = 24	M1	FT **their** x and t.
	so 24 m	F1	FT only **their** t (as long as it is +ve and is not obtained with sign error(s) e.g. –ve sign just dropped)
	or		
	Require $y = -3.6$	M1	Equating **their** y to ±3.6 or equiv. Any form.
	so $-3.6 = 8t - 4.9t^2$		
	Eliminate t between		
	$x = 12t$ and $-3.6 = 8t - 4.9t^2$	M1	Expressions in any form. Elimination must be complete
	so $0 = 3.6 + \frac{8x}{12} - \frac{4.9x^2}{144}$	A1	Accept in any form. May be implied.
	Use of formula or factorise	M1	A method for solving a 3 term quadratic to give at least 1 root. Allow **their** y and re-arrangement errors.
	+ve root is 24 so 24m	F1	FT from **their** quadratic after re-arrangement. Must be +ve.
	or		
	Methods that divide the motion into sections		
	Projection to highest point (A)		
	Highest point to level of jetty (B)		
	Level of jetty to sea (C)		
	Combination of A, B and C may be used	M1	Attempt to find times or distances for sections that give the total horizontal distance travelled
		M1	Correct method for one section to find time or distance
	(A) 0.8163.. s; 9.7959.. m: (B) 0.816…s; 9.7959.. m (C): 0.3673… s; 4.4081… m	A1	Any time or distance for a section correct
		A1	2nd time or distance correct (The two sections must not be A and B)
		A1	cao
		5	
		8	

7		mark	comment	sub
(i)	$32\cos\alpha t$	B1		1
(ii)	$32\cos\alpha \times 5 = 44.8$	M1	F **their** *x*.	
	so $160\cos\alpha = 44.8$ and $\cos\alpha = 0.28$	E1	Shown. Must see some working e.g $\cos\alpha$ = 44.8/160 or 160 $\cos\alpha$ = 44.8. If 32 × 0.28 × 5 = 44.8 seen then this needs a statement that 'hence $\cos\alpha$ = 0.28'.	2
(iii)	$\sin\alpha = 0.96$	B1	Need not be explicit e.g. accept sin(73.73…) seen.	
	either			
	$0 = (32\times0.96)^2 - 2\times9.8\times s$	M1	Allow use of '*u*' = 32, *g* = ± (10, 9.8, 9.81).	
		A1	Correct substitution.	
	s = 48.1488… so 48.1 m (3 s. f.)	A1	cao	
	or			
	Time to max height is given by 32 × 0.96 -9.8 *T* = 0 so *T* = 3.1349….	B1	Could use ½ total time of flight to the horizontal.	
	y = 32 × 0.96 *t* – 4.9 t^2	M1	Allow use of '*u*' = 32, *g* = ± (10, 9.8, 9.81) May use $s = \frac{(u+v)}{2}t$.	
	putting *t* = *T*, *y* = 48.1488 so 48.1 m (3 s. f.)	A1	ca	4
		7		